VON THÜNEN'S THEORY OF NATURAL WAGES.

II.

CRITICISMS OF THE FORMULA: NATURAL WAGES $= \sqrt{ap}$.

THUNEN'S theory of wages holds a peculiar place in German Economics. Important as the theory is, it is unmentioned in the majority of German works,—a fact which Falck and Komorzynski attribute to the abstruse mathematical character of his method. But even among the few critics that Thünen has had there is a lack of harmony concerning the correctness of his work: some accept his mathematical results in their entirety; others accept a part, and refuse the rest; while still others reject his results *in toto*.

In the following pages my purpose is to endeavor to show that those critics who have offered the strongest objections to the correctness of Thünen's results have either overlooked the limited premises from which he started, or forgotten the hypothetical nature of his conclusions. At the end I shall try to expose the error which, so far as Thünen's own work is concerned, vitiates the correctness of his formula.

1. Let us examine first the position of Falck, who was one of the last to devote a monograph to Thünen's work. The main object of Falck's dissertation is to prove the mathematical inaccuracy in the formula for natural wages. He claims that the formula $\sqrt{ap}$ "is the keystone of Thünen's whole system"; and, if you reject it, his "system loses all practical importance." *

*Falck, *Die Thunen'sche Lehre vom Bildungsgesetz des Zinsfusses und vom naturgemassen Arbeitslohn*, p. 32.

He proceeds in his argument as follows: Thünen obtained the formula $\sqrt{ap}$ from the expression $\frac{[p-(a+y)]\,y}{q\,(a+y)}$. If we represent this expression by R, then

$$R = \frac{[p-(a+y)]\,y}{q\,(a+y)} = \frac{py}{q\,(a+y)} - \frac{y}{q} = \left[\frac{p}{a+y} - 1\right]\frac{y}{q}.$$

From the last expression we find, when $y = o$, the value of R becomes o. In other words, when wages are reduced to the bare means of subsistence, the rent of the capital-producer vanishes. This fact seems to startle Falck; and, forgetting that Thünen's work has nothing to do with actual conditions, but is based upon purely hypothetical conditions, he claims that the phenomenon indicated by the expression can have no economic cause, "since we can daily convince ourselves that, as a matter of fact, in many places wages are reduced not only to the bare means of subsistence, but even below that amount, while the income of the capitalist not only does not vanish, but may increase." * He infers, therefore, that there is a mathematical error in the expression; and he claims to prove the error by the following argument: —

"The formula $\frac{[p-(a+y)]\,y}{q\,(a+y)}$ was obtained from the formula $\frac{n\,[p-(a+y)]}{\frac{nq\,(a+y)}{y}}$. The numerator denotes the rent from the farm, the denominator the number of those among whom the rent is divided. But is the y of the denominator really equivalent to the y of the numerator?" He denies that the y's are equivalent, asserting that the "y of the numerator denotes the surplus that is paid to the laborer at this particular time; but the y of the denominator denotes the surplus of wages that existed before the laying out of the farm." "Only by placing these two y's equal to each other," he claims, "has it been possible for the rent to obtain a maximum value at a definite rate of wages." †

Falck is under the impression that the rate of wages which exists before the laying out of the farm is different

* Falck, p. 34. † *Ibid.*, p. 35.

from the rate that exists after the farm is completed. As this is an error into which several critics have fallen, it will be well to show that, in supposing a number of laborers to combine in laying out a marginal farm, Thünen proceeds on the assumption that natural wages exist in the isolated state before the farm is begun, and that the supposition is introduced merely as a means of ascertaining the mathematical expression for the prevailing rate of wages. Thünen attempts to place such conditions upon the isolated state, that natural wages not only can be realized, but are realized; and his main problem is to find the mathematical expression for such wages.

Let us briefly review the chief limitations that he places upon the isolated state. He assumes that the isolated state is in a static condition; that the laborers are equal in intelligence, skill, etc.; and that perfect competition pervades the entire state. He assumes that beyond the margin of cultivation there is a limitless territory whose fertility is equal to that of lands already under cultivation, and he maintains that the rate of wages and the rate of interest existing at the margin of cultivation determine the rate of wages and rate of interest throughout the entire state. With these conditions placed upon the isolated state, Thünen claims that the mere possibility * of laborers laying out farms for themselves will compel the undertakers to pay laborers wages that will be equal to what the latter could earn by laying out farms and cultivating them on their own responsibility.

He argues (pp. 146, 147), if the undertakers should attempt to lower wages, laborers would emigrate to the margin, and begin cultivation on their own responsibility. Since, however, the number of laborers is constant, this act on their part would cause scarcity of labor in the interior of the state, which would result in a loss to the

* "Die blosse Moglichkeit fur die Arbeiter, sich in der Wildniss anzusiedeln ohne dass dies That wird," etc. *Der isolirte Staat*, Part II., p. 147.

undertakers. If, therefore, the undertakers desire to keep their laborers, they must pay them such wages as would make emigration to the margin of the state unprofitable.

Falck might claim that Thünen's supposition of a party of laborers combining to lay out a farm is an illustration of the manner in which the laborers would proceed to enforce higher wages when undertakers had reduced them below the natural limit. But this claim could not be sustained; for Thünen distinctly tells us that his investigations rest upon the supposition that the isolated state is in a static condition (*im beharrenden Zustand*, p. 146). By static condition he does not mean that the laborers are at war with their employers, trying to obtain natural wages. He means that they already receive natural wages. This follows from what he himself says concerning the static condition: "Im isolirten Staat haben wir . . . stets den endlichen Erfolg, also das erreichte Ziel, vor Augen gehabt. Mit dem erreichten Ziel tritt Ruhe und damit der beharrende Zustand ein; und hier erblicken wir Gesetzmässigkeit, während in der Uebergangsperiode Manches uns als ein unentwirrbares Chaos erscheint" (p. 35). He believes that natural wages already exist in the isolated state; and, as a means of discovering the mathematical expression for such wages, he supposes that a number of laborers, to whom it is a matter of indifference whether they labor for wages or cultivate a marginal farm on their own account, combine to lay out a farm.

We can now easily see the fallacy in Falck's objection. Falck rests his whole proof of the inaccuracy of Thünen's formula on his claim that the y of the denominator of the expression $\frac{n[p-(a+y)]}{\frac{nq(a+y)}{y}}$ has a different value from the y of the numerator. The sole reason that he offers to substantiate his claim is that the y of the denominator represents

the surplus that the laborer receives before the farm is begun, and the y of the numerator represents the surplus that he receives after the farm is completed. But this is no reason whatever why the y's are not equal. Because y represents two quantities, it by no means follows that those quantities are unequal. If the analysis that I have given of Thünen's method of procedure is correct, natural wages are assumed to exist in the isolated state both before and after the completion of the marginal farm; and consequently the y's in the above expression are equal.

2. Roscher* claims that even in the isolated state, Thünen's formula $\sqrt{ap}$ does not represent natural wages; for, he says, if labor and capital are combined in different proportions in different industries, the laborers will not be justly rewarded if all receive wages $= \sqrt{ap}$. For example, if an artist, using cheap fuel, makes valuable vases out of cheap clay, it is not just or natural that he should receive wages equal to or less than the wages of an ordinary laborer. But Professor Roscher claims that the formula $\sqrt{ap}$ would bring about this relation, because a has the same value for the artist and the laborer, and p varies with the amount of capital used. Hence, when the laborer is employed in an industry where much capital is used, "product of labor," p, will be very great; and, if his wages $= \sqrt{ap}$, he may receive more than the artist who uses only a small amount of capital.

Komorzynski† holds that Roscher is right in saying that $\sqrt{ap}$ does not represent natural wages, where labor and capital combine in different proportions in production; but he shows that Roscher's illustration is defective, since he compares laborers of different classes, an artist and an ordinary laborer, whereas Thünen's investigations are concerned only with the ordinary laborer.

* Roscher, *Geschichte der Nationalokonomik in Deutschland*, p. 896.

† Komorzynski, "Thunen's naturgemasser Arbeitslohn," *Zeitschrift fur Volkswirthschaft, Socialpolitik, und Verwaltung*, Dritter Band, 1 Heft, p. 53.

Now let us examine the argument offered by Roscher and approved by Komorzynski, that, where capital and labor combine in different proportions in production, $\sqrt{ap}$ does not represent natural wages even in the isolated state. In order to present the matter clearly, I shall use an illustration found in Thünen's own work,* where capital and labor combine in different proportions in production. In § 13 he announces the law that the price of commodities tends to conform to their cost of production, and attempts to illustrate the law by a comparison of the prices of mining and agricultural products. In the early part of his work he assumed that the mines of the isolated state lay in the neighborhood of the city; but, in order that he may illustrate his law by a comparison of the prices of silver and grain, he supposes, for the time being, that the silver mines are scattered about the state, that the last mine which is worked lies at the margin of the state, and that, further in the wilderness, mines of equal fertility to the marginal mine are found, but that they are not worked, because the product would not pay for the cost of production. This follows from the fact that the product of the marginal mine just covers the cost of production.

Now, it has already been observed that the product of the marginal farm just covers the cost of production. Then, since perfect competition pervades the isolated state, and laborers may work either at farming or mining, it follows that the wages of the farm laborer and the wages of the miner must have equal values. Hence, if we can find the wages of the former in terms of grain and the wages of the latter in terms of silver, we shall be able to find the exchange values of grain and silver at the margin of the isolated state.

Thünen's formula for wages is $a + y = \frac{p}{1+qz}$. He supposes that the rate of interest — which, of course, is the same for owners of mines and owners of farms — is five

* Part II., pp. 131, 132.

per cent. In the above formula, and, indeed, everywhere else in Thünen's work, p represents "product of labor" *in kind*. When the laborers are employed in silver mining, p represents a certain amount of silver. When they are employed in agriculture, it represents a certain amount of grain. Thünen supposes that in the case of the marginal silver mine p equals $7\frac{1}{2}$ pounds of silver, and in the case of the marginal farm p equals 240 scheffels of rye. He then says, "Since different industries require different amounts of capital, q will represent different quantities." * He supposes that in mining $q = 20$, and in agriculture $q = 12$. By making these substitutions in the formula, we find that the wages of the miner $(a + y) = \frac{7\frac{1}{2}}{1 + 20 \times \frac{1}{20}} = 3\frac{3}{4}$ pounds of silver; and the wages, of the farm laborer $(a + y) = \frac{240}{1 + 12 \times \frac{1}{20}} = 150$ scheffels of rye. $3\frac{3}{4}$ pounds of silver, therefore, has the same value as 150 scheffels of rye; and $7\frac{1}{2}$ pounds of silver, the "product of labor," p, in silver mining, has the same value as 300 scheffels of rye.

Now, Roscher and Komorzynski would say $\sqrt{ap}$ does not represent natural or just wages, because the miner would receive as wages $\sqrt{ap} = \sqrt{a \times 300}$; while the farm laborer would receive $\sqrt{ap} = \sqrt{a \times 240}$. Whatever the value of a may be, it is the same for both; and consequently the miner would receive more than the farm laborer. Upon the face of it, this argument looks sound; but yet we should scarcely expect Thünen to make this blunder after telling us that throughout the isolated state all laborers receive the same wages, and that in different industries the quantity q, the amount of capital used, is different. As a matter of fact, I think it may be proved that the error lies with Roscher and Komorzynski. They attach

* Part II., p. 131. Falck attempts to defend Thunen against the criticism of Roscher, and says "Der Herr Verfasser [Roscher] scheint hier vergessen zu haben, dass die Arbeiter des isolirten Staats Alle mit dem gleichen Capital ausgerustet sind und dass daher ein verschiedenes Quotverhaltniss geradezu undenkbar ist" (p. 55). The quotation above from Thunen, together with his illustration of different amounts of capital used in silver mining and in agriculture, show the absurd position that Falck has taken.

an entirely wrong meaning to p when p occurs in the formula $\sqrt{ap}$. In the formula $\sqrt{ap}$, p means "product of labor" in agriculture at the margin of the isolated state, and in the formula $\sqrt{ap}$ it never means anything else.

In the beginning of his work Thünen gives a general definition of p as the "product of labor" of one man; that is, the product to be shared between the laborer and the owner of the capital which he uses. He proceeds with his work, and bases his investigations at times upon actual conditions, and at times upon the assumed conditions of the isolated state. In his investigations based upon actual conditions he develops the formula $z = \frac{p-(a+y)}{q(a+y)}$; but, as his object is to find the expression for the natural wages of the isolated state, he tells us that in its present form the expression $z = \frac{p-(a+y)}{q(a+y)}$ is of no use to him. But why? Because p does not represent a constant quantity, "but rises and falls with q. y and z, in turn, depend upon p. Hence p, y, and z are functions of q." * He then definitely and distinctly states the problem before him. "The problem is then," he says, "to find the value of p, y, and z for a given value of q." †

After announcing his problem, he immediately leaves his investigations based upon actual conditions, and goes to the isolated state. He then shows that, if the laborers are to be deterred from laying out marginal farms, this equation must exist $(a+y)+q(a+y)z=p$, where p equals the "product of labor" in agriculture on the marginal lands. "Here," he says, "a, p, and q are determinate, y and z indeterminate" (p. 141). With these quantities assumed as known, he proceeds with his investigations, and ends by declaring that natural wages $=\sqrt{ap}$. But what does p mean here? Evidently, it has but one meaning, the "product of labor" in agriculture at the margin of the isolated state.‡

* Part II., p. 139. † *Ibid.*

‡ Misconception with regard to the meaning of p as that quantity occurs in the formula $\sqrt{ap}$ has been a source of numerous errors, and it is important that

3. Komorzynski takes a peculiar position in his criticism of Thünen. Thünen tells us he considers his isolating method the most valuable part of his work;* but Komorzynski insists that the isolated state, with all its appurtenances, is an unnecessary part of Thünen's work, and serves merely to obscure his investigations and confuse his critics. Thünen believes that the formula for natural wages can be found only in one way; namely, by means of the conditions placed upon the isolated state.† But Komorzynski claims that Thünen unwittingly develops the formula in two ways: that the second method of obtaining the formula has nothing to do with the isolated state, but is based upon actual conditions; and, when the two methods are divested of all unnecessary suppositions, the

the point made above should be understood. Thunen's purpose is merely to investigate, under the favorable conditions of the isolated state, the influence of free land upon wages. His position is that, at the margin of cultivation in the isolated state, the determination of the rate of wages and the rate of interest will be in accord with the best interests of the laborer, and that the rate of wages and the rate of interest at the margin determine the rates throughout the state. If Thünen attempted to show anything, it was that wages at the margin equal $\sqrt{ap}$, where p equals "product of labor" in agriculture; and his claim is that the same wages will exist throughout the isolated state. "Wir behaupten, dass der an der Grenze des isolirten Staats sich bildende Arbeitslohn und Zinsfuss normirend fur den ganzen Staat ist" (p. 142).

Thünen's assumption that the mines of the isolated state are situated near the city (Part I., p. 1) confirms the idea that it was his purpose to investigate under favorable conditions the influence of free land upon wages; for, if the mines had been so placed that the one whose product just covered its cost of production was situated at the margin of the state, and that a number of equally fertile mines were left untouched in the wilderness, then, instead of the laborers combining to lay out a farm on the margin, they would unite to work a mine. We might then make suppositions concerning the latter combination similar to those that Thunen made concerning the former combination; and, if we should use capital letters where Thunen used small ones, we should find, by following Thunen's method, that the wages of each miner would be $\sqrt{AP}$. But $A = a$, and P is greater in value than p, because more capital is used in the production of P than is used in the production of p. Hence $\sqrt{AP}$ is greater than $\sqrt{ap}$; and, consequently, it would be more profitable for the laborers to engage in mining than in agriculture. Thunen saw what the difficulty would be if he supposed the mines to be placed in this way; and for this reason he was careful, in making the illustration I have given in the text, to state that his supposition concerning the position of the mines was only temporary.

*Part I., p. xix †Part II, p. 26.

ascertainment of the formula is found to rest upon two conditions which are common to the two methods.

The second method of obtaining the formula $\sqrt{ap}$ is, according to Komorzynski, as follows: In the part of his work based upon actual conditions Thünen develops the formula $z = \frac{p-(a+y)}{q(a+y)}$, and states that it expresses the rate of interest. In another part of his work he states that it is to the advantage of the laborer to have wages and interest bear such relation to each other that the laborer will receive the maximum income from his surplus when that surplus is placed at interest. Now, Komorzynski argues, since the surplus is y, and the rate of interest $z = \frac{p-(a+y)}{q(a+y)}$, the interests of the laborer will be subserved when yz, which is equal to $\frac{y\,[p-(a+y)]}{q(a+y)}$, obtains a maximum value. This takes place when $(a+y) = \sqrt{ap}$. He therefore concludes that Thünen's suppositions concerning the isolated state are all unnecessary, and all that is needed to prove that $\sqrt{ap}$ represents the best rate of wages for the laborer is to prove: —

A. That $z = \frac{p-(a+y)}{q(a+y)}$ expresses a general formula for interest.

B. That it is to the advantage of the laborer to have wages and interest bear such relation to each other that the laborer will receive a maximum income from his surplus when that surplus is placed at interest.

Why Thünen did not proceed in the manner indicated I shall consider later on. At present we shall consider Komorzynski's attempt to prove an error in the development of the formula $\sqrt{ap}$ on the ground that the conditions A and B are unsustainable.

A. He claims that the subtle error which vitiates the formula for natural wages is that $z = \frac{p-(a+y)}{q(a+y)}$ is not a general formula for the rate of interest. To fulfil the conditions of a general formula for the rate of interest, it must express the same rate of interest for all industries.

But Komorzynski says the above formula is not capable of doing this; for "wages $(a+y)$ and interest z are by means of Thünen's formula $z=\frac{p-(a+y)}{q(a+y)}$ brought into a relation which, in turn, varies with the changing quantities p and q. The quantity p, the surplus value of the product over the value of the capital consumed in production, and likewise the quantity q, . . . are different in different forms of production. From this it follows that the relation which, according to the formula, exists between the rate of interest z and the rate of wages $(a+y)$, will be different in different forms of production." * In other words, since the quantities p and q are different in different forms of production, he infers that the relation of $(a+y)$ and z expressed by $z=\frac{p-(a+y)}{q(a+y)}$ will be different in different forms of production; and therefore, since the formula does not express the same rate of interest for all industries, it is not a general formula for the rate of interest.

The error in the argument can best be exposed by means of the example that Komorzynski gives to illustrate his argument. The illustration is intended to show why the formula does not represent the general rate of interest. He says: "If the rate of interest is 5 per cent. and the rate of wages 400 florins, then, in three different forms of production, these equations may exist: —

I. $\frac{5}{100}=\frac{2{,}500-400}{105\times 400}$, where $p=2{,}500$ and $q=105$;
II. $\frac{5}{100}=\frac{1{,}200-400}{40\times 400}$, where $p=1{,}200$ and $q=40$;
III. $\frac{5}{100}=\frac{460-400}{3\times 400}$, where $p=460$ and $q=3$;

But, if $(a+y)$ should rise from 400 to 450, then the following unequal rates of interest would result: —

I. $\frac{4.33}{100}=\frac{2{,}500-450}{105\times 450}$; II. $\frac{4\,16}{100}=\frac{1{,}200-450}{40\times 450}$; III. $\frac{.741}{100}=\frac{460-450}{3\times 450}$." †

* Komorzynski, p. 58.

† *Ibid.*, p. 59, note. There is a slight error in the original which I have corrected in the quotation given above. Where $\frac{.741}{100}$ occurs above, $\frac{7.41}{100}$ is found in the original.

This example, he claims, illustrates that the formula is not a general formula for the rate of interest, or, better, a formula for the general rate of interest, but that it represents the relation between wages and interest in specific industries.

The fallacy in the argument may be readily shown. Komorzynski has overlooked Thünen's definition of p. In his illustration Komorzynski makes p represent, in each of the three cases, a definite number of florins. This is evident from the fact that in each case he subtracts wages, expressed in terms or florins, from p. Furthermore, when he supposes wages to rise from 400 to 450 florins, he assumes that in each of the three cases p retains its value in terms of florins. In his illustration and throughout his whole article * he assumes that p represents a definite *value*. But Thünen distinctly defines p as "product of labor"; that is, "product of labor" in kind (pp. 80, 167). When laborers are employed in silver mining, p is expressed in terms of silver (p. 131); and, when they are employed in agriculture, p is expressed in terms of grain (p. 131). If, then, p means "product of labor" in kind, Komorzynski cannot assume that the value of p, or the price of p, expressed in florins, remains constant when the rate of wages changes. This error renders his objections useless; for if, after the rise of wages, the demands of the community require the continuance of production in groups I., II., III., the formula $z = \frac{p-(a+y)}{q(a+y)}$ (provided there is no other objection to the formula than that which Komorzynski offers) may still represent equal rates of interest for the three groups if p, remaining constant in quantity as "product of labor" in kind, may change its value or change its price in terms of florins.

B. Komorzynski also attempts to prove defective Thünen's supposition that it is to the advantage of the laborer

* For example, p. 58, "Die Grosse p, der Wertuberschuss des Productes," etc.; p. 55, "Hier bedeutet p den Tauschwert (erlangbaren Verkaufspreis) des Productes," etc.

to have wages and interest bear such a relation to each other that the laborer will receive a maximum income from his surplus when that surplus is placed at interest.* In brief, he claims that it is not to the interest of the laborer that *yz* should attain a maximum value.

His objection may be thus stated: The interests of laborers as to the relation of wages and interest vary according as they have saved during many or during few years. If they have saved during many years, they desire to have wages low and interest high. If, on the other hand, they are just beginning to save, they desire a higher rate of wages and a lower rate of interest. Hence there is no definite relation of wages and interest that, under all circumstances, is the best relation for the laborers. Concerning Thünen's supposition that it is to the advantage of the laborer to have *yz* attain a maximum value, Komorzynski says that it is based on the arbitrary assumption that the laborers save only during one year, and that all laborers have only an amount of capital equal to *y*.

This objection has much in its favor; but, before considering it, we must put Thünen in the right light. In introducing Komorzynski's criticism, it was stated that his position is peculiar, since he holds that, while Thünen believed he could obtain his formula for natural wages only in one way,—by means of the conditions of the isolated state,—he unwittingly develops it in two ways; that, when the two methods are disembarrassed of all superfluous suppositions, the ascertainment of the formula is found to rest upon two conditions, which are common to the two methods; and that the ascertainment of the formula by the second method is based, not upon the hypothetical conditions of the isolated state, but upon actual conditions. By the logic of his position, therefore,

* This question is also discussed by Knapp, G. F., *Zur Prufung der Untersuchungen Thunen's uber Lohn und Zinsfuss im isolirten Staate*, pp. 18–26; Roscher, *Geschichte*, etc., p. 896; Schmidt, C., *Der naturliche Arbeitslohn*, pp. 34–37.

he is forced to charge Thünen with holding that it is to the advantage of the laborer, under actual conditions, to have wages and interest bear such relation to each other that he will receive the maximum income from his surplus when that surplus is placed at interest. This charge cannot be proved. Thünen's hypothesis that there is a definite relation of wages and interest which is most desirable for the laborer is based upon the supposition that there is a direct interdependence between wages and interest. But he cannot be charged with claiming a direct interdependence between wages and interest under actual conditions,* for nowhere in his investigations based upon actual conditions does he make such a supposition. So far, however, as concerns the isolated state,— a state in a static condition, with constant capital, with constant population having constant wants and constant methods of production,— his claim of an interdependence of wages and interest cannot be denied.

Now, when the above objection is considered as directed towards Thünen's method of developing the formula on the basis of the isolated state, it has the merit, not of proving his investigations false, but of showing them to be painfully contracted and incomplete. In a state where there is a direct interdependence of wages and interest, it is true that, when laborers possess different amounts of capital, those who possess much will desire a different relation of wages and interest from those who possess little. It is also true that, when laborers have saved during many years, they desire a different relation of wages and interest from what they desired when they were just beginning to save. But, after we have admitted

* Komorzynski's error grows out of his misunderstanding of the formula $z=\frac{p-(a+y)}{q(a+y}$, as that formula occurs in § 13. *If* Thünen claimed that the formula represents a general formula for interest under actual circumstances, and *if* he intended p to represent a constant price, as Komorzynski understands it, then we might charge him with holding that there is a direct interdependence of wages and interest under actual circumstances.

all this, we cannot deny that the laborers who are just beginning to save will be interested in having the relation of wages and interest such that they will obtain a maximum income from their surplus when that surplus is placed at interest. If, then, y equals the surplus, and z the rate of interest, we cannot deny that, when laborers begin to save, it will be to their advantage to have yz obtain a maximum value. Komorzynski's argument does not prove an error in Thünen's work: it merely shows its incompleteness.

4. I shall now attempt to point out a fallacy in Thünen's reasoning that vitiates his formula; and, in order to expose this fallacy, I must briefly review the chief points in § 13, that repeatedly misinterpreted section of Thünen's work. As I have shown in the early part of this article, Thünen's prime object in § 13 is to find means by which he can reduce to terms of labor the co-operation of capital in production. He proceeds to do this by saying, if an amount of capital Q is divided by the year's wages of a laborer $(a+y)$, we shall find "how large the capital is, expressed in years' labor of a laboring family" ("Wie gross das Kapital in Jahresarbeiten einer Arbeiterfamilie ausgedrückt ist," p. 124). Although we see what Thünen means to say, yet his words do not express his meaning. By dividing capital by wages, he does not obtain an expression for capital in terms of years' labor of a laboring family, but in terms of wages. Further on in the same section (p. 128) he repeats the same idea by saying, in effect, that capital may be reduced to terms of labor by dividing the amount of capital by wages. This inaccuracy in the use of words continues throughout the chapter.

In continuing his work, he supposes that $\frac{Q}{a+y} = nq$, or $Q = nq\,(a+y)$. If the capital $nq\,(a+y)$ is used in a productive process where n laborers are employed, then, assuming that the "product of labor" of each man equals p, and the wages of each equal $(a+y)$, Thünen holds

that the rate of interest is expressed by the formula $z = \frac{n[p-(a+y)]}{nq(a+y)} = \frac{p-(a+y)}{q(a+y)}$. By a manipulation of this formula he claims to prove that "z is the factor by means of which the relation of the efficiency of capital to that of human labor is expressed"; and hence he concludes, "We are herewith placed into position to reduce to terms of labor the co-operation of capital in the production of a commodity." Here he sums up in two sentences the result of his investigations in § 13. Notice, however, that above he spoke of reducing *capital* to terms of labor, while here he speaks of reducing the *co-operation of capital* to terms of labor,— two entirely different things, which we must keep distinctly separated. In a moment we shall find how Falck has fallen into error by confusing these two processes.

Now let us see what use Thünen makes of these results. To find the expression for natural wages, he resorts to the case of a number of laborers combining to lay out a marginal farm. One of the quantities that he uses to obtain his formula is nq, and the manner in which he obtains that quantity he states in these words: Suppose "the laying out of the farm required the year's labor of nq men. . . . Unquestionably, in order to provide a new farm, is needed not only labor, but also the use of capital; (but) according to § 13, we can reduce the co-operation of capital to terms of labor, and thus express the costs of laying out the farm entirely in terms of labor." *

What does Thünen mean by saying, according to § 13, the co-operation of capital may be reduced to terms of labor? Knapp understands him to signify that the co-operation of capital can be reduced to terms of labor by means of the rate of interest, and he states that such a reduction is impossible. This interpretation of Thünen's meaning I should urge as correct, and should base the claim (1) upon the general meaning of § 13, which is

* Part II., p. 152.

expressed in these sentences: "z is the factor by means of which the relation of the efficiency of capital to that of human labor is expressed," and "We are herewith placed into position to reduce to terms of labor the co-operation of capital in the production of a commodity"; (2) upon Thünen's use of words. In the paragraph under discussion he says, "According to § 13, we can reduce the co-operation of capital [*die Mitwirkung des Kapitals*] to terms of labor"; and in the latter of the two sentences just quoted he uses the identical expression, the co-operation of capital (*die Mitwirkung des Kapitals*).

Falck, however, takes exception to Knapp's interpretation, and states his objection by saying that in the first place, by dividing the value of capital by the value of a year's wages, the reduction of which Thünen speaks is possible; and in the second place, according to Thünen, the reduction is not effected by means of the rate of interest. Curiously enough, he informs the reader that, by referring to § 13 of Thünen's work, he may convince himself of the validity of his objection to Knapp's criticism. The truth of the matter is that Falck not only misinterprets both Thünen and Knapp, but he also misquotes Knapp. That he misinterprets those authors is evident from the fact that they speak of reducing the co-operation of capital to terms of labor, and Falck speaks of reducing capital to terms of labor. That he misquotes Knapp can be readily seen by comparing page 16 of Knapp with page 23 of Falck.*

If the interpretation that I have given of § 13 is correct,

*Knapp, p. 16: "Thünen bemerkt . . . das ein Grenzgut nicht nur durch Verwendung von Löhnen hergestellt werden kann; er glaubt aber . . . man brauche bloss 'die Mitwirkung des Capitals auf Arbeit zu reduciren,' so erreiche man das Gewünschte; diese Reduction, selbst wenn sie möglich wäre, geschieht nach Thünen nur durch den Zinsfuss, und da der noch zu finden ist, so bleibt also die Schwierigkeit ungelöst."

Falck, p. 23: "Schliesslich sagt Knapp, . . . 'Diese Reduktion (des Capitals auf Arbeit), selbst wenn sie möglich wäre,'" etc.

Knapp is speaking of the reduction of the co-operation of capital, and Falck quotes him as speaking of the reduction of capital.

it is easy to expose a fallacy in Thünen's reasoning that vitiates the formula $\sqrt{ap}$. Thünen's purpose in his whole work is to find mathematical expressions for the natural rate of interest and the natural rate of wages. The method by which he does this is first to find a formula expressing the interdependence of wages and interest in the isolated state. This formula $z = \frac{p-(a+y)}{q(a+y)}$ we shall call formula A. In this formula all the quantities are known except y and z. In order to find the values of y and z, he next attempts to find an independent expression for y, or what is the same thing, since a is known, an independent expression for $(a+y)$; and by substituting for $(a+y)$ in formula A obtain the value of z. The formula that enables him to find the independent expression for $(a+y)$ is $\frac{n[p-(a+y)]}{\frac{nq(a+y)}{y}}$. In this formula, which we shall call formula B, all the quantities are assumed as known excepting y. But how did Thünen obtain the quantity nq? He says: Suppose "the laying out of the farm required the year's labor of nq men. . . . Unquestionably, in order to provide a new farm, is needed not only labor, but also the use of capital; (but) according to § 13, we can reduce the co-operation of capital to terms of labor, and thus express the costs of laying out the farm entirely in terms of labor." When we refer to § 13 to see how the reduction is to be performed, we find that it is done by means of the rate of interest. The fallacy in the argument is evident. Thünen's whole procedure is a mere begging of the question. His problem is to find the values of y and z in formula A; and, to solve the problem, he undertakes to find an independent expression for $(a+y)$ by means of formula B, and by substituting for $(a+y)$ in formula A obtain the value of z. But, in order to get the quantity nq in formula B, he assumes that z is known. If, however, z is known, then, according to formula A, y is known. Thünen undertakes to find the value of the unknown quantities y and z; and, in attempting to solve the problem,

he uses the very quantities that he wants to find as known quantities.

While this error renders useless the formula $\sqrt{ap}$, yet it enables us to see why Thünen did not proceed to obtain the formula in the manner indicated by Komorzynski, and shows us the continuity in his work. In the part of his work based upon actual conditions (§ 13) he obtains the formula $z = \frac{n[p-(a+y)]}{nq(a+y)} = \frac{p-(a+y)}{q(a+y)}$; and in another part he states that it is to the advantage of the laborer to have the relation of wages and interest such that he can obtain the maximum income from his surplus when that surplus is placed at interest,— that is to say, it is to the advantage of the laborer to have yz obtain a maximum value. Now, Komorzynski holds that in these few facts we have the data with which to obtain the formula for natural wages, and that Thünen need not have carried his work farther; for, he says, if it is to the advantage of the laborer to have yz obtain a maximum value, then, since $z = \frac{p-(a+y)}{q(a+y)}$, his interests will be subserved when $\frac{y[p-(a+y)]}{q(a+y)}$ obtains a maximum value, which is the case when $(a+y) = \sqrt{ap}$. But Komorzynski assumes that the value of q in the above expression is known; and, in making this assumption, he has fallen into error. In § 13 the denominator of the expression for interest is obtained by dividing a definite amount of capital, Q, by the rate of wages. $\frac{Q}{a+y} = nq$ or $Q = nq(a+y)$. Hence nq is unknown as long as $(a+y)$ is unknown; and, when nq is known, $(a+y)$ is known, because their product is the definite quantity Q. Since, then, the value of q depends upon the value of $(a+y)$, it is an error to attempt to find the value of the unknown quantity $(a+y)$ in the above expression by assuming that q is known.

Thünen's method of obtaining the formula leads us to believe that he foresaw this difficulty. His sole object in trying to reduce the co-operation of capital in production to terms of labor was to enable him to proceed with his

work by considering nq a known quantity. When he attempted to find the expression for natural wages by assuming that a number of laborers combined to lay out a marginal farm, he did not begin by assuming that the value of the farm equalled Q, and by dividing that quantity by the rate of wages obtain nq; but he began by supposing that nq equalled the number of men required to lay out the farm. Knapp and Komorzynski* notice that he changed his method of obtaining nq, and, without seeing his purpose, charge him with inconsistency. By following his work, however, we find that he had a definite purpose in view; for, assuming that nq is known, he obtained an expression that is identical in form with the above expression from which Komorzynski obtains the formula $\sqrt{ap}$, but differs from it, in that nq, according to Thünen, is a known quantity.

Thünen's theory is valuable,† because it marks a decided reaction against the teachings of the classical economists, and yet at the same time avoids the extravagant doctrines of the socialists. His specific contribution to the theory of natural wages does not consist in his mathematical formulas, nor, indeed, in any positive conclusions that he obtains, but rather in his designation of the factors that must be considered in any scientific theory of natural or just wages. What these factors are can best be seen by contrasting Thünen's theory with that of the classical economists.

1. While the classical economists treated labor throughout as a mere commodity, Thünen regards the laborer as a *man*, and considers his wages as the means of satisfying

*Knapp, pp. 15, 25. Komorzynski, pp. 55, 56.

†It must be remembered that the first edition of *Der naturgemasse Arbeitslohn* appeared in 1850. For valuable suggestions concerning Thunen's contribution to Natural Wages, compare Schumacher's *Ueber J. H. v. Thunen's Gesetz vom naturgemassen Arbeitslohn*, pp. 18, 19, Mithoff (in Schonberg, *Handbuch*, Band 1), p. 640; Schmidt, pp. 2, 16.

his wants. The work that he undertakes is to find the wages that are agreeable to the nature (*naturgemàss*) and to the destiny of man.

2. While the classical economists regarded solely the operation of natural law, Thünen considers the equity in the case. He believes that natural wages exist in the isolated state when these two conditions are realized, (*a*) when the laborer receives the same income from his surplus when that surplus is placed at interest as the capital-producer receives from his surplus when that surplus is embodied in a marginal farm; (*b*) when the laborer receives the maximum income from his surplus.* Here Thünen makes a crude attempt to find an equitable basis for the division of the product of labor between the laborers and the owners of capital invested in concrete forms.

3. While the classical economists considered only the requirements of the laborer as limited by his surroundings, and disregarded the product of labor, Thünen holds that there can be no scientific theory of wages that does not make wages depend upon product. The fundamental idea in the formula $\sqrt{ap}$ is that wages must vary with the product.

A scientific theory of natural wages must regard the laborer as a man, consider the rights of the laborer and of the capitalist, and make the wages of the laborer depend upon his product.

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* These two characteristics of natural wages are definitely stated, p. 204.